To check that $\mathbb{E}\left[\sum_{n=1}^{\infty} X_n 1_{\xi_n > n_3}\right] = \sum_{n=1}^{\infty} \mathbb{E}[X_n 1_{\xi_n > n_3}]$ in the proof of Wald's equation:

Recall that a sufficient condition is

$$\sum_{n=1}^{\infty} \mathbb{E}\left[|X_n 1_{\xi_n > n_3}|\right] < \infty$$

(this can be thought of as Fubini's theorem, but more fundamentally it is a consequence of the dominated convergence theorem).

Now,

$$\sum_{n=1}^{\infty} \mathbb{E}\left[|X_n 1_{\xi_n > n_3}|\right]$$

$$= \sum_{n=1}^{\infty} \mathbb{E}[|X_n| 1_{\xi_n > n_3}]$$

$$= \sum_{n=1}^{\infty} \mathbb{E}[|X_n|] \mathbb{E}[1_{\xi_n > n_3}] \text{ by independence}$$

$$= \mathbb{E}[|X|] \sum_{n=1}^{\infty} \mathbb{P}[N > n]$$

$$= \mathbb{E}[|X|] \mathbb{E}[N] < \infty \text{ by assumption}$$

(formally, this is where we use the requirement that $\mathbb{E}[N] < \infty$)