The exam problems will test your ability in two main areas:

1. Constructing simple code or describing what a bit of code is doing
   - manipulation of data frames
   - control structures: for, while, if-else
   - using vectorization instead of loops

2. Being able to understand and apply the algorithms taught in class
   - Inversion method for random number generation
   - Rejection sampling for random number generation
   - Importance sampling
   - Newton-Raphson

**Example 1:** Suppose \( X \) contains a 10-by-10 data frame. Describe what each of the following lines of code does:

\[
X[c(1,4,7),c(2:5)]
\]

\[
X[,-c(6:8)]
\]

\[
X[abs(X[,7]) < 1, ]
\]

**Example 2:** Write a short program to calculate the smallest \( K \) such that

\[
\sum_{n=1}^{K} n^{3/2} \leq 50
\]

**Example 3:** Choose which of the responses is approximately what this program print at the end and explain why:

\[
X <- \text{matrix}(\text{rexp}(20000, \text{rate}=1/3), 1000, 20)
\]

\[
M <- \text{apply}(X, 1, \text{mean})
\]

\[
\text{print( mean(M), var(M) )}
\]

The options are

[1] 3 9

[1] 3 0.45

[1] 0.333 0.111

[1] 0.15 0.45

**Example 4:** Describe in terms of a conditional expectation what the following program estimates:
X <- rnorm(10000)
X <- X[which( (X > 0) & (X < 2) )]
mean(X)

**Example 5:** Write a program which does the same as the following without using a loop

X <- rnorm(10000)
for(j in 1:10000) if(X[j] < 0) X[j] <- 0 else X[j] <- sqrt( X[j] )

**Example 6:** Consider rejection sampling from the standard normal density,

\[ p(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \]

using the Cauchy distribution

\[ g(x) = \frac{1}{\pi(1 + x^2)} \]

as the trial distribution. If the first uniform generated was \( U = 0.632 \) and the first candidate draw generated was \( X = 1.36 \), would you accept or reject this candidate? Justify your answer.

*Note:* You may take it for granted that \( M = \sup_x p(x)/g(x) \approx 1.52 \).

**Example 7:** Suppose you have a distribution with density

\[ p(x) = \lambda x^{-\lambda-1} \]

where \( x \geq 1 \) and \( \lambda \geq 2 \). Write a one line function based on the inversion method to generate \( n \) samples from the distribution where \( \lambda \) is an input the the function.

What line could you type to estimate \( E(X^{2.736}) \) if \( X \) has the distribution described above with \( \lambda = 3 \)?

**Example 8:** Suppose you wanted to estimate the integral

\[ \int_{-3}^{3} e^{-|x|} dx \]

by monte carlo. Write two short programs to do this by

1. viewing this as an expectation against the Uniform\((-3,3)\) density
2. importance sampling with \( N(0,1) \) as your trial density

Which of these do you expect to provide a smaller standard error for the integral approximation?

**Example 9:** Consider optimization of the function \( f(x) = \exp(-x^2 + 3x + 4) \). Suppose your start value is \( x_0 = 1 \). Where would you be after a single Newton-Raphson update?