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Some Solutions to Homework 3

Instructor: Robert Keener
GSI: Runlong Tang
Statistics Department, U-M

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Notes

• We only provide solutions to some problems.
• For each problem, just one possible solution is provided. You solutions can be different.
1 Solution to Problem 2.55 on Page 54:

(a). Denote

\[ E_1 = \{ \text{a hand contains the ace and king of spades.} \} \]
\[ E_2 = \{ \text{a hand contains the ace and king of hearts.} \} \]
\[ E_3 = \{ \text{a hand contains the ace and king of diamonds.} \} \]
\[ E_4 = \{ \text{a hand contains the ace and king of clubs.} \} \]

By the inclusion-exclusion identity, we have

\[
P(\bigcup_{i=1}^{4} E_i) = \sum_{i=1}^{4} P(E_i) - \sum_{i<j} P(E_i E_j) + \sum_{i<j<k} P(E_i E_j E_k) - P(E_1 E_2 E_3 E_4)
\]

\[= 4P(E_1) - \binom{4}{2} P(E_1 E_2) + \binom{4}{3} P(E_1 E_2 E_3) - P(E_1 E_2 E_3 E_4).\]

Next, let’s calculate the above probabilities. We have

\[ P(E_1) = \frac{\binom{50}{11}}{\binom{52}{13}}, \quad P(E_1) = \frac{\binom{48}{9}}{\binom{52}{13}}, \quad P(E_1) = \frac{\binom{46}{7}}{\binom{52}{13}}, \quad P(E_1) = \frac{\binom{44}{5}}{\binom{52}{13}}.\]

Finally, we can compute \(P(\bigcup_{i=1}^{4} E_i) \approx 0.219.\)

(b). The approach is similar to (a). The probability of interest is about 0.0342.

2 Solution to Problem 3.14 on Page 102:

(b). There are six cases and the probabilities of them are the same to the result of (a).

3 Solution to Problem 3.28 on Page 104:

(a). Denote \(A\) as the event that the first ace is the 20th card and \(B\) as the event that the following card is ace of spades. Then, we are interested in finding \(P(B|A) = P(AB)/P(A)\). Thus, we only need to calculate \(P(A)\) and \(P(AB)\) separately. The probability is \(3/128\).

(b). Similar to (a). The probability is \(39/1536\).
4 Solution to Problem 3.1 on Page 110:

By the definition of conditional probability, we have

\[ P(AB|A) = \frac{P(AB)}{P(A)}, \quad P(AB|A \cup B) = \frac{P(AB)}{P(A \cup B)}. \]

Since \( A \subset A \cup B \), we have \( 0 < P(A) \leq P(A \cup B) \). Thus we have

\[ P(AB|A) \geq P(AB|A \cup B). \]

5 Solution to Problem 3.8 on Page 111:

(a). From \( P(A|C) > P(B|C) \), we have \( P(AC) > P(BC) \). Similarly, from \( P(A|C^c) > P(B|C^c) \), we have \( P(AC^c) > P(BC^c) \). Thus, we have \( P(AC) + P(AC^c) > P(BC) + P(BC^c) \), which is just \( P(A) > P(B) \).

(b). According to the hint, We can calculate \( P(A|C) = \frac{1}{3} \) and \( P(A|C^c) = \frac{5}{33} \). Thus, \( P(A|C) > P(A|C^c) \). Similarly, we can also calculate \( P(B|C) = \frac{1}{3} \) and \( P(B|C^c) = \frac{5}{33} \) and have \( P(B|C) > P(B|C^c) \). This means those two conditions hold for this specific example.

Next, we need show that \( P(AB|C) > P(AB|C^c) \) does not hold. Actually, \( P(AB|C) = 0 \). Since a probability must be nonnegative (in \([0,1]\)), \( 0 > P(AB|C^c) \) can not happen, which finished the proof.

(In fact, \( P(AB|C^c) = \frac{1}{33} \).)