Statistics 547 Assignment 1

Due in class on Monday, February 11.

1. Under geometric distributions with $p = .001, .01, .05$, compute the exact probabilities of observing a value within 1, 2 and 3 standard deviations of the mean (it will be useful to know that the variance of the geometric distribution is $(1 - p)/p^2$). Compare these to the probabilities obtained from the normal approximation.

2. For $p = .001, .01, .05, 1, 5, 9$ and $n = 200, 500, 1000, 10000$, let $\lambda(n, p) = np$. For each $(n, p)$, compute the exact binomial probability that $S_n$ lies in the interval $\lambda^*(n, p) \pm 5$, where $\lambda^*(n, p)$ is $\lambda(n, p)$ rounded to the nearest integer. Then compute the Poisson approximation to the same probability.

3. Generate a random iid sequence of 5,000 bases with uniform probabilities. Compute the number of occurrences $N$ of the triplet AGT. Repeat this process 300 times so that you have 300 independent observations of $N$: $N_1, \ldots, N_{300}$. Compute the mean and variance of $N$ under the binomial model and its Poisson approximation, then compare the sample mean and variance of the $N_i$ to these values.