Math Stat 425
Practice Exam

Instructions: Show your work and explain your reasoning carefully.

1. A committee of size three is to be selected from a group of six Democrats, five Independents, and four Republicans. What is the probability that the Democrats have a majority on the committee.
   Ans: .341

2. A drawer contains two red sox, two white sox, two blue socks, and two green socks. If four socks are selected at random, what is the probability that at least two are of the same color.
   Ans: .771

3. In Problem 1, what is the conditional probability that the Democrats have a majority, given that there is at least one Republican on the committee?
   Ans: .207

4. Small University has two students, three professors, and four deans. On any given day the students show up for class with probability .9 each, the professors show up for work with probability .6 each, and the deans show up for work with probability .3 each. Classes are held iff at least one member from each group shows up. Assuming independence, what is the probability that classes are held?
   Ans: .704

5. In the previous problem, which would increase the probability that classes are held more: admitting an additional student, hiring and additional professor, or hiring an additional dean?
   Ans: Hiring an additional dean.

6. A test for a rare disease has a false positive rate of 2% and a false negative rate of 1%. Suppose that .5% of the population have the disease. If a person takes the test as part of a routine physical exam, what is the probability that it will come back positive (indicating the disease to be present)? What is the conditional probability that the person has the disease, given that the test is positive?
   Ans: .0249; .1992

7. Let $A_1, A_2, \cdots$ be independent events for which $P(A_k) = 1/(k+1)$ for $k = 1, 2, \cdots$. Find $P(\bigcup_{k=1}^{\infty} A_k)$.
   Ans: 1

8. Two tickets are drawn without replacement from a box containing one ticket labelled “one,” two tickets labelled “two,” three tickets labelled “three,” and four tickets labelled “four.” Let $X$ be the absolute difference between the labels on the two tickets drawn. Find the probability mass function, mean, and standard deviation of $X$.
   Ans: $\mu = 1.2; \sigma = .884$

9. On a true-false examination, a student knows the answer to any given question with probability .6 and guesses otherwise. If there are ten questions, what is probability that he/she gets at least eight of them right?
   Ans: .678

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10. The bottle capping machine at the ABC Brewery malfunctions with probability .0005 each time that it operates. If 10,000 bottles are produced in a day, what is the probability that there are exactly five defective caps; at least five?
Ans: .1755; .5595

11. Two evenly matched teams play a series of games until one team has won at least four. Let \( X \) be the number of games required to decide the series. Find the probability mass function and mean of \( X \).
Ans: \( \mu = 5 \) 13/16

12. A random variable \( X \) has a density of the form
\[
f(x) = \begin{cases} c(1 - |x|) & \text{if } -1 \leq x \leq 1 \\ 0 & \text{if otherwise,} \end{cases}
\]
where \( c \) is a constant. Find \( P[0 < X < \frac{1}{2}] \), \( E[X] \), and the standard deviation of \( X \).
Ans: 3/8; 1/3; 1/\( \sqrt{6} \).

13. A room is lighted by five globes that burn independently for exponentially distributed life times with failure rate \( \lambda = .02 \) per hour. What is the probability that at least one of the globes is still burning after 100 hours? What is the probability that at least three are still burning after 50 hours?
Ans: .517; .264

14. Let
\[
F(x) = e^{-e^{-x}}
\]
for \(-\infty < x < \infty\). Show that \( F \) is a distribution function and find its median.
Ans: .3665

15. Suppose that quantitative GRE Scores are normally distributed with mean \( \mu = 540 \) and standard deviation \( \sigma = 120 \). Find the probability that a student scores more than 750. What is the 95th percentile of the distribution of scores?
Ans: .0401; 737.4