Math Stat 425 Problem Set 2
Due: October 3, 2007

Instructions. Do all of Problems 1-13 and as many of the Additional Suggested Problems as possible. Write out the solutions to any five of the even numbered problems (2, 4, 6, 8, 10, 12) and turn them in on the due date. Show your work and explain your reasoning.

1. Do Problem 3.70 (seventh edition) from the text. The answer in the book.

2. In Problem 3.70, suppose that the queen has had only one healthy prince (without hemophilia). What is the (conditional) probability that a second prince will have hemophilia?

3. If a balanced die is rolled 12 times, what is the probability that each of the six faces appears at least once?
   Ans: .4378.

4. What is the probability that all of the four suits are represented in a hand of seven cards (a combination of seven cards from a standard deck)?
   Hint: Consider the complementary event.

5. A committee of size three is to be selected from a group of ten men and ten women. What is the conditional probability that both sexes are represented, given that there is at least one man on the committee?
   Ans: .909

6. In Problem 5, suppose that the committee size is four. What is the conditional probability that the committee contains two men and two women, given that it contains at least one person of each sex?

7. In a certain court, cases are decided by a single judge. Suppose that the judge finds an innocent person guilty with probability .2 and finds a guilty person guilty with probability .9. Suppose also that 60% of defendants are guilty. What proportion of defendants are convicted (found guilty)? What proportion of people convicted are actually guilty?
   Ans: .62 and .871

8. On a True-False examination, a student knows the answer to questions with probability .70 and guesses otherwise. Given that he/she answered a questions correctly, what is the probability that he/she knows the answer?

9. Do Problem 3.66(a) in the text when \( p_i = .9, \ i = 1, \cdots 5 \).
   Ans: .8675.

10. Do Problem 3.66(b) when \( p_i = .9, \ i = 1, \cdots, 5 \).

11. Let \( A_1, A_2, \cdots \) be independent events for which \( P(A_k) = 1/(k + 1)^2 \) for each \( k \); and let \( B_n = \bigcup_{k=1}^{n} A_k \) and \( B_\infty = \bigcup_{k=1}^{\infty} A_k \) for \( n = 1, 2, \cdots \). Find \( P(B_n) \) when \( n = 10 \).
Ans: 5/11.

12. In Problem 11, find the probability of $P(B_n)$ as a function of $n$ and also the probability of $B_{-\infty}$. Explain the relevance of the Monotone Sequences Theorem here.

13. A six-sided die is so loaded that the probability of getting $k$ spots is proportional to $k$. If the die is rolled twice, what is the probability that larger of the number spots appearing on the two roles is equal to five?

Ans: .241