Math Stat 425 Problem Set 3
Due October 19, 2005

Instructions. Do all of Problems 1-13 and as many of the Additional Suggested Problems as possible. Write out the solutions to any five the even numbered Problems (2,4,6,8,10,12). Show your work and explain your reasoning. Students may work in pairs if they wish. In such cases, only one problem set should be handed in with the names of both students on it.

1. Let $X$ be the absolute difference between the number of spots that appear when two balanced (six-sided) dice are tossed (larger less smaller). Find the probability mass function, mean, and variance of $X$.
   Ans: $\mu = 1.944$ and $\sigma^2 = 2.054$.

2. A group of three men and two women are ranked from best to worst, 1 to 5. Suppose that all $5!$ ranking are equally likely. Let $X$ be the best rank attained by a woman. Find the probability mass function, mean, and variance of $X$.

3. Do Problem 4.42 in the text.
   Ans: in the text.

4. Do Problem 4.43 in the text.

5. There were 1095 marriages in a certain town last year. Let $X$ denote the number of couples whose birthdays fall on the same day of the year. Find $P[X = 3]$, $P[X \geq 3]$, and $E(X)$ What assumptions are you making?
   Ans: .224, .577, and 3.

6. The probability of winning a prize in a lottery is 1/100. If a person plays every day for a year, what is the probability that he/she wins exactly three prizes; at least three?

7. A random variable $X$ has a density of the form $f(x) = c(1 - x^2)^2$ for $-1 \leq x \leq 1$ and $f(x) = 0$ for other values of $x$, where $c$ is a constant. Find the distribution function, mean, and variance of $X$
   Ans: $F(x) = 1/2 + (15x - 10x^3 + 3x^5)/16$ for $-1 \leq x \leq 1$; $\mu = 0$; and $\sigma^2 = 1/7$.

8. A random variable $X$ has a density of the form $f(x) = c|x|/(1 + x^2)^4$ for $-\infty < x < \infty$, where $c$ is a constant. Find the distribution function, mean, and variance of $X$

9. If $X$ is a positive random variable with failure rate $\lambda(t) = t$, $0 \leq t < \infty$, what are the distribution function and density of $X$?
   Ans: $f(t) = t \exp(-t^2/2)$ and $F(t) = 1 - \exp(-t^2/2)$ for $0 \leq t < \infty$.

10. If $X$ is a positive random variable with failure rate $\lambda(t) = 2/(1 + t)$, $0 \leq t < \infty$, what are the distribution function and density of $X$.

11. The ABC Bolt company produces bolts of nominal diameter 1”. In fact, the diameter of a typical bolt is a normally distributed random variable $X$ with mean $\mu = 1”$ and standard deviation $\sigma = .002”$. A bolt meets specifications if its diameter is between .997” and 1.003”. What percentage of the bolts produced meet specifications?
   Ans: 86.6.
12. In the Land of Oz, systolic blood pressures of college age men are normally distributed with mean $\mu = 120$ and standard deviation $\sigma = 10$. What percentage of college age men have blood pressures between 115 and 135? What percentage have blood pressure over 135?

13. A woman plays 100 independent games of roulette, winning $1 on each play with probability $9/19$ and losing $1 otherwise. Let $X$ denote her net winnings (possibly negative). Approximate the probability that $X > 0$.

$Ans$: .265.

Additional Suggested Problems. Chapter 4: 14, 20, 38, 40, 46, and Theoretical Exercises 4.14, 4.16, and 4.18. Chapter 5: 7, 11, 15, 16, 19, 31, 32, and 40. (The numbering is the same in the 6th and 7th editions.)