Midterm 1: Stat 426.

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**Announcement:** The total number of points is 25 but the maximum you can score is 20.

(1) Let $X$ be an Exponential(1) random variable and define $Y = \lfloor X \rfloor + 1$ where $\lfloor X \rfloor$ is the largest integer not exceeding $X$. Then $Y$ is a discrete random variable.

   Compute the probability mass function of $Y$. Can you identify the distribution of $Y$ as something you have seen before? What is $E(Y)$? (7 points)

(2) A, B and C play the following game: A fair die is rolled and A wins if 1 or 2 turn up, B wins if 3 or 4 turn up and C wins if 5 or 6 turn up. They keep on playing till B wins for the first time. Find the chance that they have to keep playing $k$ games till B wins. What is the average number of games that they need to play? (8 points)

(3) Let $X$ and $Y$ be independent Exponential (1) random variables. Let $U = X$ and $V = X/Y$. Compute the joint density of $(U, V)$ and hence, find the marginal densities of $U$ and $V$. (10 points)