1. A poker hand of five cards is dealt from a well shuffled standard deck of 52 cards. Find the chance that all four suits are represented.

2. A carpet store gets half of its inventory from manufacturer A and half from manufacturer B. The number of flaws in a roll of carpet from manufacturer A is a Poisson random variable with mean $\lambda = 1$, and the number of flaws in a roll of carpet from manufacturer B is a Poisson random variable with mean $\lambda = 2$. Suppose a roll of carpet from the store selected at random has 5 flaws. What is the chance that this roll of carpet was supplied by manufacturer A.

3. A die is rolled twice. Let A be the event that the maximum of the two rolls is a 5 and let B be the event that the sum of the two rolls is 7. Determine whether A and B are independent. Give probability calculations that support your answer.

4. In a certain state, the number of people who win a government run lottery on a given day is a Poisson random variable with mean $\lambda = 1$. Find the chance that during a given week there are exactly 3 days with no winners.

5. A random variable $X$ has cumulative distribution function

$$F_X(x) = \begin{cases} 0, & \text{for } x \leq 0; \\ \sin(x), & \text{for } x \in (0, \pi/2); \\ 1, & \text{for } x \geq \pi/2. \end{cases}$$

Find the mean $\mu = E[X]$, and compute the chance that $X$ exceeds its mean $\mu$.

6. Suppose X and Y have joint density

$$f(x, y) = \begin{cases} 3x, & \text{if } 0 < y < x < 1; \\ 0, & \text{otherwise}. \end{cases}$$

Find the marginal density of Y and find the covariance between $X$ and $Y$.

7. Let $X$ be an exponential variable with parameter $\lambda = 1$ and let $Y$ be an exponential variable with parameter $\lambda = 2$. Let $Z = \min\{X, Y\}$. Assuming that $X$ and $Y$ are independent, find the cumulative distribution function of $Z$. Hint: Using the independence of $X$ and $Y$ you can compute $P(Z > z)$ without using the joint density of $X$ and $Y$.

8. Let $X$ and $Y$ be independent exponential random variables, both with parameter $\lambda = 1$. Use Chebyshev’s inequality to bound $P(|Z| \geq 5)$.

9. You have just joined a line to buy tickets for a basketball game. Unfortunately there are 100 people in line ahead of you. Suppose the times it takes to process the people ahead of you are i.i.d. exponential random variables with mean $\mu = 1/2$ minute. Find the (approximate) chance that it will take longer than one hour for you to arrive at the head of the line.

10. Four cards are dealt from a well shuffled standard deck of 52 playing cards.

   a. Find the chance that the four cards are from different suits.

   b. Suppose we are told that exactly two of the cards are aces. Find the (conditional) chance that the four cards are from different suits.

11. As a purchasing agent for a small construction company, you buy a shipment of 200 parts from a supplier. You know that this supplier has two machines used to make these parts, one old
and one new. There is a 50% chance your shipment came from the new machine and a 50% chance it came from the old machine. The new machine is better—each item from the new machine has a 1% chance of being defective, while the chance of a defective item for the old machine is 2%. In the parts below, feel free to use any approximations (normal or Poisson) that should lead to an accurate answer.

a. Find the chance your shipment contains exactly one defective item.

b. Suppose your shipment contains one defective item. What is the conditional probability the new machine was used to make the parts in your shipment.

c. You inspect the first 100 items in your shipment and find exactly one item which is defective. Find the conditional chance of one or more defectives in the other half of your shipment.

12. A random variable $X$ has density 

$$f_X(x) = \frac{1}{2}e^{-|x|}.$$ 

a. Find the mean and variance of $X$.

b. Use Chebyshev’s inequality to find an upper bound for $P(|X| \geq 5)$.

c. Find $P(|X| \geq 5)$.

13. Suppose $X_1, X_2, X_3$ have a multinomial distribution with $p_1 = p_2 = p_3 = 1/3$ and $n = 2$ trials.

a. Find $E[1/(1 + X_1 + 2X_2)]$.

b. Find the chance that $X_1X_2 + X_3$ is an even number.

14. The random variables $X$ and $Y$ have joint density 

$$f(x, y) = \begin{cases} x + y, & \text{for } 0 < x < 1 \text{ and } 0 < y < 1; \\ 0, & \text{otherwise.} \end{cases}$$

a. Find the marginal density of $X$.

b. Find the chance $Y$ is less than half $X$.

c. Find $E[XY]$.

15. The number of points a certain basketball team scores in any given game has mean 98 and variance 100. Find the chance this team scores more than 5000 total points in a 50 game season.

16. Let $X_1, X_2, X_3, X_4, X_5$ be independent variables each uniformly distributed on the interval $(0,1)$. For $a \in (0, 1)$, find $P(\min\{X_1, \ldots, X_5\} < a)$.

17. A coin is tossed repeatedly. Let $X$ be the number of heads during the first two tosses, let $Y$ be the number of tosses needed to get two heads, and let $p(x, y)$ be the joint probability mass function of $X$ and $Y$. Find $p(1,4)$. 

2